Enhanced Quantum Nonlinearities in a Two-Mode Optomechanical System

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In cavity optomechanics, nanomechanical motion couples to a localized optical mode. The regime of single-photon strong coupling is reached when the optical shift induced by a single phonon becomes comparable to the cavity linewidth. We consider a setup in this regime comprising two optical modes and one mechanical mode. For mechanical frequencies nearly resonant to the optical level splitting, we find the photon-phonon and the photon-photon interactions to be significantly enhanced. In addition to dispersive phonon detection in a novel regime, this offers the prospect of optomechanical photon measurement. We study these quantum nondemolition detection processes using both analytical and numerical approaches.

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Introduction.—By coupling mechanical resonators to the light of optical cavities, the field of optomechanics [1] aims at observing quantum mechanical behavior of macroscopic systems. New architectures and progress in fabrication pave the way towards realizing strong coupling at the single-photon level in optomechanical systems [2–7]. This development has stimulated several theoretical works that analyze the generic optomechanical system, i.e., a single optical mode coupled to a single mechanical mode, in the regime of strong coupling. Nonclassical effects are found in the dynamics of the mechanical resonator [8–10] and the statistics of the light field [9,11,12] if the photon-phonon coupling rate g_0 becomes comparable to both the decay rate of the cavity κ and the mechanical oscillation frequency Ω.

In this Letter, we show how an optomechanical setup consisting of two optical modes coupled to a mechanical resonator [13–15] can be brought into a novel regime that significantly enhances the size of the quantum nonlinearity. We derive an effective Hamiltonian of the system that captures the regime of strong single-photon optomechanical coupling $g_0/\kappa \gtrsim 1$ and large mechanical frequencies. The difference between optical level splitting and mechanical frequency, $\delta\Omega = 2J - \Omega$, appears as a crucial parameter. It enters the coupling rate $g_0^2/\delta\Omega$ that characterizes the coherent interaction among photons and between photons and phonons. If this dispersive optical frequency shift exceeds the cavity decay rate, one enters what we will call the strong dispersive coupling regime: $g_0^2 \gtrsim \kappa\delta\Omega$. Since $\delta\Omega$ can be made much smaller than Ω, this condition is easier to achieve than the corresponding one for the generic optomechanical system, $g_0^2 \gtrsim \kappa\Omega$. This is relevant, in particular, because optomechanical systems have by now reached the regime of large mechanical frequencies [5–7].

As a first application of the enhanced phonon-photon interaction we investigate the possibility of a quantum nondemolition (QND) detection of the phonon number. A measurement of this kind has been proposed in a pioneering work by Thompson *et al.* [13] for a setup where a dielectric membrane is placed inside an optical cavity. Subsequently, this QND scheme [16–19] and other features of such a two mode system [20–25] have been studied in detail. An increase of the nonlinear coupling by making use of the full spectrum of cavity modes has been demonstrated in [26–28]. However, the analysis has so far been restricted to cases where the influence of individual photons is weak. Furthermore, it was assumed that the mechanical and optical time scales separate. Hence, the previous analysis did not capture the enhancement of the optomechanical nonlinearity, which, as we show below, results in an increased read-out rate.

As a completely new feature of optomechanical systems, our effective description reveals strong photon-photon interaction for mechanical frequencies comparable to the optical mode splitting and opens up the possibility of a QND measurement of the photon number. The two mode optomechanical system can therefore be assigned to a larger class of optical systems whose ultimate goal is the realization of QND photon detection on the level of single quanta [29].

In our analysis of the phonon and photon Fock state measurements, we discuss the limitations due to quantum noise and confirm our predictions by numerical simulations of the dissipative quantum dynamics.

Model.—We consider an optomechanical setup consisting of two optical modes ($a_\pm$, frequencies $\omega_\pm$) and one mechanical mode (b, frequency Ω) that is described by a Hamiltonian $H = H_0 + H_{\rm int} + H_{\rm drive} + H_{\rm diss}$, where

$$H_0 = \hbar\omega_- a_-^\dagger a_- + \hbar\omega_+ a_+^\dagger a_+ + \hbar\Omega b^\dagger b, \qquad (1)$$

$$H_{\rm int} = -\hbar g_0 (b^\dagger + b)(a_+^\dagger a_- + a_-^\dagger a_+), \qquad (2)$$

and $H_{\rm drive} = \hbar\alpha_\pm(e^{i\omega_{L\pm}t}a_\pm + {\rm H.c.})$. The optomechanical coupling rate is denoted by g_0, and both optical modes are

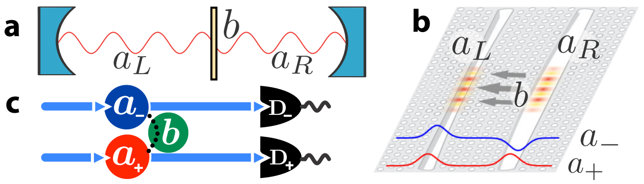

FIG. 1 (color online). (a) and (b) Example implementations of the double cavity setup for enhanced quantum nonlinearities: membrane in the middle (a) and optomechanical crystal setup (b). (c) Scheme depicting the mechanical mode b and the optical modes $a_\pm$. For the photon and phonon detection applications discussed in this Letter, the cavities are assumed to be driven by independent laser sources and the transmitted signal is measured by photodetectors $D_\pm$.

pumped by laser sources at rates $\alpha_\pm$. The optical cavities are characterized by photon decay rates into the reflection ($\kappa_{\pm,r}$) and the transmission channel ($\kappa_{\pm,t}$) with $\kappa_\pm = \kappa_{\pm,r} + \kappa_{\pm,t}$. We assume the transmitted signal from each of the modes to be filtered and measured independently using a photodetector [Fig. 1(c)]. The mechanical resonator couples to a thermal bath at a rate Γ with a bath occupation given by $n_{\rm th}$. In the following, we assume the ratio of bath temperature to mechanical frequency to be small enough such that the oscillator is close to the ground state.

A Hamiltonian of this kind is found in the "membrane in the middle"-setup [13], in coupled microtoroid resonators [14] and in optomechanical crystals [15]. The optical modes $a_\pm$ constitute normal modes $a_\pm = (a_L \pm a_R)/\sqrt{2}$, where $a_{L,R}$ denotes geometrically distinct modes with an original Hamiltonian $\tilde{H} = \tilde{H}_0 + \tilde{H}_{\rm int}$, where $\tilde{H}_{\rm int} = -\hbar J(a_L^\dagger a_R + {\rm H.c.}) - \hbar g_0(b^\dagger + b)(a_L^\dagger a_L - a_R^\dagger a_R)$ and $\tilde{H}_0 = \hbar\omega(a_L^\dagger a_L + a_R^\dagger a_R) + \hbar\Omega b^\dagger b$. The frequency splitting of the normal modes is given by the photon tunneling rate J, $\omega_- - \omega_+ = 2J$.

In the approach of Refs. [13,16–18], the optical resonances are calculated as $\omega \pm \sqrt{J^2 + (g_0\tilde{x})^2} \approx \omega_\pm \pm (g_0^2/2J)\tilde{x}^2$ [Fig. 2(a)], where $J \gg g_0\tilde{x}$ is assumed and the mechanical displacement $\tilde{x} = b^\dagger + b$ is treated as a quasistatic variable (in the sense of the Born-Oppenheimer approximation, with photons playing the role of electrons). This approach therefore has to fail if the optical frequency splitting and the mechanical frequency become comparable.

Effective description.—The effect of the optomechanical interaction to first order in g_0 can be described in the following picture. A photon initially placed in the left (or right) cavity mode starts oscillating between the left and right part of the cavity at a frequency $2J$: $(a_L^\dagger a_L - a_R^\dagger a_R)(t) \approx a_+^\dagger(0)a_-(0)e^{-2iJt} + {\rm H.c.} + \mathcal{O}(g_0^2)$. Accordingly, the radiation pressure force $F = g_0\sqrt{2\hbar m\Omega}(a_+^\dagger a_- + a_-^\dagger a_+)$ varies sinusoidally in time. This force drives mechanical oscillations $x_{\rm osc} = F/[m(\Omega^2 - 4J^2)]$ and $p_{\rm osc} = [1/(\Omega^2 - 4J^2)]F'(t)$, where $F'(t) = -2iJ(a_+^\dagger a_- - a_-^\dagger a_+)(t)$.

To take these elementary dynamics into account, we shift the oscillator by x_{osc} and p_{osc} via a unitary transformation

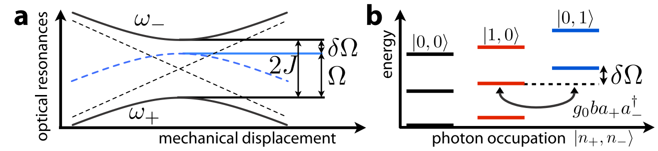

FIG. 2 (color online). (a) Optical resonances vs mechanical displacement. For $\delta\Omega = 2J - \Omega \ll \Omega$, J, the regime of enhanced quantum nonlinearity is reached. (b) Energy level scheme of the double cavity optomechanical system. The most relevant second-order transition process is indicated.

$H_{\rm eff} = e^{iS}(H_0 + H_{\rm int})e^{-iS}$, with $S = x_{\rm osc}p/\hbar + p_{\rm osc}x/\hbar$. This procedure eliminates the interaction to first order in g_0 and results in an effective Hamiltonian

$$H_{\rm eff} = H_0 + \hbar\frac{g_0^2}{2}\left(\frac{1}{2J-\Omega} + \frac{1}{2J+\Omega}\right)\left(n_- - n_+\right)\left(b^\dagger + b\right)^2 + \hbar\frac{g_0^2}{2}\left(\frac{1}{2J-\Omega} - \frac{1}{2J+\Omega}\right)\left(a_+^\dagger a_- + a_+ a_-^\dagger\right)^2, \qquad (3)$$

where $n_\pm = a_\pm^\dagger a_\pm$ and where we disregard terms of order $g_0^3/\delta\Omega^2$. For vanishing tunnel coupling J, the unitary transformation reduces to a shift of the mechanical position due to a static radiation pressure force. In this case the effective Hamiltonian is given by $H_0 - \hbar g_0^2/\Omega(a_L^\dagger a_L - a_R^\dagger a_R)^2$ in correspondence to the "polaron transformation" for the single-mode setup [9,11,30,31]. The most interesting regime is entered if the mechanical frequency becomes comparable to the optical splitting, i.e., $\delta\Omega = 2J - \Omega \ll J, \Omega$:

$$H_{\rm eff} = H_0 + \hbar\frac{g_0^2}{\delta\Omega}(n_+n_- + n_- + n_-n_b - n_+n_b), \qquad (4)$$

where $n_b = b^\dagger b$ and where we neglect terms of the order $g_0^2/(2J + \Omega)$ and rapidly rotating terms like $b^{\dagger 2}$, $(a_+^\dagger a_-)^2$.

Phonon detection.—The effective Hamiltonian of Eq. (3) enables us to discuss optomechanical QND phonon detection in its most general form, going beyond previous discussions [13,16–18]. The optical frequencies are shifted by $\mp g_0^2\{[1/(2J - \Omega)] + [1/(2J + \Omega)]\}n_b$, and in the limit $\Omega \ll J$ the result of Ref. [16] is recovered. However, for mechanical frequencies comparable to the optical splitting, i.e., $\delta\Omega = 2J - \Omega \ll 2J$, the frequency shift per phonon $\delta\omega = g_0^2/\delta\Omega$ is greatly enhanced. We stress that the enhancement of the frequency shift is observable even in the weak coupling regime $g_0 \ll \kappa_\pm$, where the cavity modes have to be strongly driven [13,18]. In the following, however, we focus on the regime where both $\Omega \approx 2J$ and $g_0 \gtrsim \kappa_\pm$.

The protocol for detecting the phonon number is to pump one of the optical modes (here a_+) with a laser at frequency ω_{L+} and measure the transmitted signal using a photodetector (D_+). The second mode (a_-) is undriven, playing the role of an idle spectator (though it will become important for dissipative processes). We first study the spectrum of the detection mode a_+, i.e., the photon number $\bar{n}_+$ as a function of detuning $\omega_+ - \omega_{L+}$. In steady state,

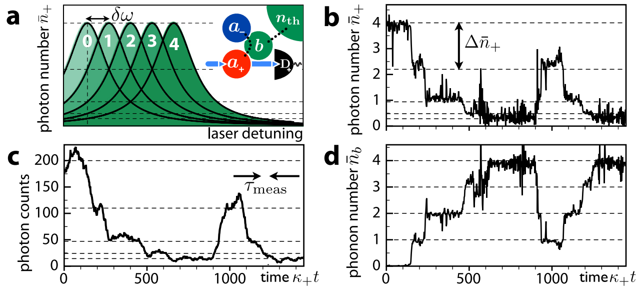

FIG. 3 (color online). Phonon detection in the weak dispersive coupling regime $\delta\omega = g_0^2/\delta\Omega < \kappa_+$, for single-photon strong coupling $g_0/\kappa_+ = 3$: (a) Schematic illustration of the resonances of the detection mode corresponding to phonon number states 0,1,2,3,4. (b) and (d) Quantum trajectories of the photon number in the detection mode, $\bar{n}_+ = \langle a_+^\dagger a_+\rangle$ (b), and the phonon number, $\bar{n}_b = \langle b^\dagger b\rangle$ (d) from a numerical simulation of the stochastic master equation. $\delta\Omega = 20\kappa_+$, $n_{\rm th} = 2$, $\Gamma = 10^{-3}\kappa_+$, $\alpha_+ = \kappa_+$, $\omega_{L+} = \omega_+$, $\kappa_- = 10^{-2}\kappa_+$, $\kappa_{\pm,t} = 0.9\kappa_\pm$. (c) Photon counts recorded at the photodetector D_+ within an interval $[t - \tau_{\rm meas}, t]$ with $\tau_{\rm meas} = 50\kappa_+^{-1}$.

the spectrum consists of several resonances with spacing $\delta\omega$ corresponding to different phonon number states. In a situation where the optical frequency shift per phonon $\delta\omega$ is smaller than the cavity linewidth κ_+, the resonances overlap [Fig. 3(a)]. In the following section, we discuss this "weak dispersive coupling" regime. The strong dispersive regime is also relevant, and we will come back to it when discussing photon measurements. The time evolution of the mechanical state can be monitored by pumping the detection mode at fixed detuning and recording the photon counts at the detector during an interval $\tau_{\rm meas}$. A quantum jump in the phonon number changes the number of intracavity photons by $\Delta\bar{n}_+$ and, accordingly, the number of detected photons by $\kappa_{+,t}\Delta\bar{n}_+\tau_{\rm meas}$. The shift in photon number can be estimated as $\Delta\bar{n}_+ \approx \bar{n}_+\delta\omega/\kappa_+$ disregarding a prefactor that depends on the detuning. The measurement time $\tau_{\rm meas}$ has to be chosen large enough, such that the measured signal exceeds the photon number uncertainty [32], i.e.,

$$\tau_{\rm meas} > \frac{\kappa_+^2/\kappa_{+,t}}{\delta\omega^2\bar{n}_+}. \qquad (5)$$

On the other hand, the measurement time has to be smaller than the lifetime of a phonon Fock state, which is governed by thermal fluctuations at rate $\Gamma_{\rm th}$ and by decoherence induced via the optical modes at rate $\Gamma_{\rm ind}$:

$$\max(\Gamma_{\rm th}, \Gamma_{\rm ind})\tau_{\rm meas} < 1. \qquad (6)$$

The thermalization rate of the phonon state $\bar{n}_b$ is given by $\Gamma_{\rm th} = \Gamma[(n_{\rm th} + 1)\bar{n}_b + n_{\rm th}(\bar{n}_b + 1)]$ in the uncoupled system. The major contribution to $\Gamma_{\rm ind}$ stems from the process where a phonon is annihilated while a photon tunnels from the a_+ to the a_- mode and decays. A calculation according to Fermi's "golden rule" yields $\Gamma_{\rm ind} \approx g_0^2\bar{n}_+\bar{n}_b\kappa_-/\delta\Omega^2$. It follows that single-photon strong coupling, i.e., $g_0^2 > \kappa_+\kappa_-$, is required to obtain a signal to noise ratio bigger than one, as has already been shown by Ref. [17] for the limiting case of small mechanical frequencies $\Omega \ll J$. We note that a phonon measurement using the a_- mode for detection can be described analogously, the main qualitative difference being that the cavity-induced decoherence processes excite phonons.

To simulate the envisaged QND phonon measurement, we employ the Lindblad master equation for the system's density matrix ρ, $\frac{d}{dt}\rho = -i[H, \rho]/\hbar + \sum \mathcal{D}[c_i]\rho + \sum \mathcal{D}[d_i]\rho$ where $\mathcal{D}[A]\rho = A\rho A^\dagger - \frac{1}{2}A^\dagger A\rho - \frac{1}{2}\rho A^\dagger A$. The unobserved channels (c_i) are the coupling to the thermal environment with $c_1 = \sqrt{\Gamma(n_{\rm th} + 1)}b$ and $c_2 = \sqrt{\Gamma n_{\rm th}}b^\dagger$, and the photon decay into the reflection channels $c_{3,4} = \sqrt{\kappa_{\pm,r}}a_\pm$, while the transmission channels $d_{1,2} = \sqrt{\kappa_{\pm,t}}a_\pm$ are under observation. We unravel the time evolution into quantum jumps [33] $\rho(t + dt) = d_i\rho(t)d_i^\dagger/\langle d_i^\dagger d_i\rangle(t)$ that occur with probability $p_i(t) = \langle d_i^\dagger d_i\rangle(t)dt$ and are interpreted as detection events at $D_\pm$, and into the deterministic part $\rho(t + dt) = \rho(t) - (i[H, \rho(t)]/\hbar - \sum_i \mathcal{D}[c_i]\rho(t) + \sum_i\{d_i^\dagger d_i/2, \rho(t)\})dt$ plus subsequent normalization. Figures 3(b)–3(d) show trajectories from such a simulation. The phonon number jumps between the Fock states 0 and 4, driven by thermal fluctuations [Fig. 3(d)]. The photon number in the detection mode follows the time evolution of the mechanical mode [Fig. 3(b)]. Thus, by monitoring the photon counts at the photodetector [Fig. 3(c)] a QND measurement of the phonon number is achieved. In contrast to earlier numerical analysis [18], our results apply to the general case of a two-sided cavity and thereby confirm the limits imposed by quantum noise [17].

Photon detection.—As a novel feature of the system, we identify the dispersive photon-photon interaction in the effective Hamiltonian (4). Here we demonstrate the prospects of a QND measurement of the photon number $\bar{n}_+$ using the a_- mode for detection. The roles of the two optical modes are chosen as to suppress the influence of unwanted transitions from the a_- mode to the energetically lower-lying a_+ mode. Both modes are driven independently by a laser and the data from the photodetector D_- is used to extract the information about the photon number $\bar{n}_+$. We assume that the detection mode has a lower finesse than the signal mode, $\kappa_- \gg \kappa_+$, such that a sufficiently large number of photons arrives at the detector D_- while the state of a_+ is only weakly perturbed by the photons in a_-.

In the weak dispersive coupling regime, $g_0^2 < \kappa_-\delta\Omega$, we find a required measurement time of

$$\tau_{\rm meas} > \frac{\kappa_-^2/\kappa_{-,t}}{\delta\omega^2\bar{n}_-}, \qquad (7)$$

in analogy to the case of phonon detection. To detect the photon state $\bar{n}_+$ within its lifetime it is also required that

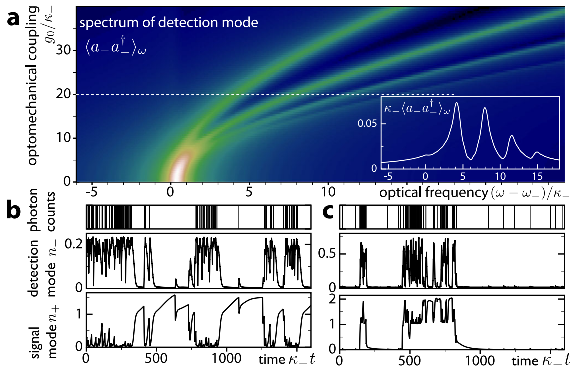

FIG. 4 (color online). (a) Spectrum of the detection mode, $\langle a_- a_-^\dagger \rangle_\omega = \int e^{i\omega\tau} \langle a_-(t+\tau) a_-(t)^\dagger \rangle d\tau$, in the presence of a strongly driven signal mode, $\bar{n}_+ = 1$. With increasing optomechanical coupling rate g_0, the splitting between the resonance peaks grows like $\delta\omega = g_0^2/\delta\Omega$. The inset shows the spectrum for $g_0 = 20\kappa_-$ (cut indicated in main figure). (b) and (c) Quantum trajectories for the detection mode driven at (b) the zero-photon resonance, $\omega_{L-} - \omega_- = \delta\omega$, and (c) at the one-photon resonance, $\omega_{L-} - \omega_- = 2\delta\omega$, showing anticorrelation and correlation between signal and detection modes. $g_0 = 20\kappa_-$, $\delta\Omega = 100\kappa_-$, $\kappa_+ = 10^{-2}\kappa_-$, $\kappa_{\pm,t} = 0.9\kappa_\pm$, $\omega_{L+} = \omega_+$, $\alpha_{L-} = \kappa_-/4$, $n_{\rm th} = 0$, $\Gamma = \kappa_-$.

$\tau_{\rm meas} < 1/\bar{n}_+\kappa_+$. Moreover, the measurement would be spoiled if a phonon were to be excited during the measurement time since a_- measures $n_+ + n_b$. We therefore demand that the thermalization rate $\Gamma_{\rm th}$ and the rate for the optically induced heating process, given by $g_0^2 \bar{n}_- \kappa_+/\delta\Omega^2$, are smaller than the measurement rate $\tau_{\rm meas}^{-1}$. From the latter condition it follows that single-photon strong coupling, $g_0^2/\kappa_+\kappa_- > 1$, is also required for an undisturbed photon detection.

In the strong dispersive regime, $g_0^2 > \kappa_-\delta\Omega$, a strong projective measurement of the photon number (or analogously the phonon number) can be performed as illustrated in Fig. 4. The spectrum of the detection mode a_-, i.e., the intensity as a function of laser detuning, shows well-resolved resonances with spacing $\delta\omega$ [Fig. 4(a)]. The weights of the peaks correspond to the photon number distribution of the signal mode. This is in close analogy to the results of Refs. [34,35]. The quantum trajectory simulations [Figs. 4(b) and 4(c)] reveal strong measurement induced backaction leading to (anti)correlation between signal and detection mode. Whenever the photodetector D_- registers photons from the detection mode, the state of the signal mode a_+ is projected into the zero- or one-photon Fock state depending on the detuning of the detection mode. This projection leads to a disruption of the coherent evolution of the signal mode as is clearly visible in Figs. 4(b) and 4(c). Note that for $\tau_{\rm meas}^{-1} > \kappa_+$, this kind of measurement backaction affects the quantum evolution significantly. It can be shown that the photons impinging on the signal mode a_- from the coherent laser source tend to be prevented from entering the cavity due to the continuous observation of the photon number inside the cavity, a manifestation of the quantum Zeno effect [36].

Experimental prospects.—Single-photon strong coupling, i.e., $g_0 > \kappa$, has been demonstrated in optomechanical systems where the mechanical element is a cloud of cold atoms [2–4]. In principle, currently available setups of this kind are extensible to a two-mode design by making use of the spectrum of transverse cavity modes [26]. Reaching $\Omega \approx 2J$ would additionally require larger trapping frequencies, $\Omega > \kappa$.

A number of optomechanical systems exhibit large mechanical frequencies of a few GHz, and $\Omega \approx 2J$ has been demonstrated [14,15,27]. Single-photon strong coupling, however, is yet to be reached in solid-state systems. The current record is achieved in optomechanical crystal setups, $g_0 \approx 0.007\kappa \approx 2\pi \times 1$ MHz [37]. Utilizing nanoslots [38] to enhance the local optical field in such structures offers the prospect of coupling rates above 10 MHz. Advances in design, fabrication and material properties are expected to lead to high-quality optical cavities with $\kappa/2\pi \approx 10$ MHz [39,40]. These developments, taken together, should make $g_0 > \kappa$ attainable.

Conclusions and outlook.—The results presented here demonstrate how the design flexibility of photonic crystals and other optomechanical systems can be exploited to significantly enhance nonlinear coupling rates. Besides the dispersive QND measurement schemes, one may think of studies of optomechanical quantum many-body effects in arrays or of further applications in quantum information processing (see also the related work by Stannigel *et al.* [41]). The coherent Kerr-type interaction introduced here can form the basis for an all-optical switch and makes it possible to engineer a quantum phase gate (based on the conditional phase shift) for photonic or phononic qubits. In addition, the mechanical mode can serve as a quantum memory [42], and optomechanical interactions yield a quantum interface between solid-state, optical, and atomic qubits [15,43]. The combination of these ingredients will make optomechanical systems a promising integrated platform for quantum repeaters and general hybrid quantum networks.

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