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# Dissipative optomechanical squeezing of light

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**Abstract**
We discuss a simple yet surprisingly effective mechanism which allows the generation of squeezed output light from an optomechanical cavity. In contrast to the well known mechanism of 'ponderomotive squeezing', our scheme generates squeezed output light by explicitly using the dissipative nature of the mechanical resonator. We show that our scheme has many advantages over ponderomotive squeezing; in particular, it is far more effective in the good cavity limit commonly used in experiments. Furthermore, the squeezing generated in our approach can be directly used to enhance the intrinsic measurement sensitivity of the optomechanical cavity; one does not have to feed the squeezed light into a separate measurement device. As our scheme is very general, it could also e.g. be implemented using superconducting circuits.

Keywords: squeezing, optomechanics, reservoir engineering, coherent feedback, measurements, measurement sensitivity, dissipative squeezing

## 1. Introduction

Among the simplest kinds of non-classical light is squeezed light, where fluctuations in one quadrature of the optical amplitude drop below the level of vacuum noise. Such light is interesting from both fundamental and practical points of view. Squeezed light can be used to improve the measurement sensitivity in applications ranging from gravitational wave detection

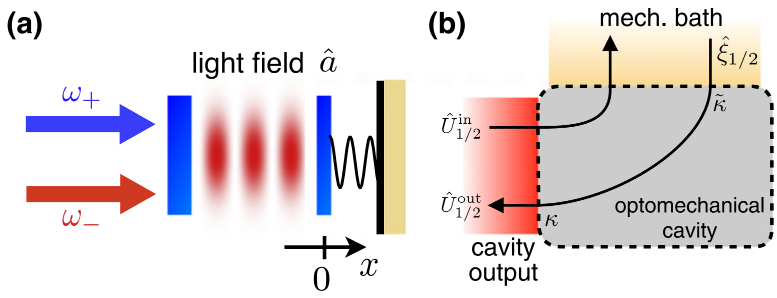


**Figure 1.** (a) Basic setup for dissipative generation of squeezed output light: an optomechanical cavity driven by two lasers on the red and blue mechanical sideband. By carefully tuning the amplitudes of the lasers, strong squeezing is possible. (b) Schematic showing the basic idea of the dissipative squeezing mechanism. Optical vacuum fluctuations (red) entering the optomechanical cavity at a rate $\kappa$ are perfectly absorbed by the mechanical resonator. At the same time, the damped mechanical resonator acts as an effective squeezed dissipative bath for cavity photons (even though the mechanical resonator itself is in a *thermal* state). The effectively squeezed mechanical noise $\hat{\xi}$ (coupled to the cavity at a rate $\tilde{\kappa}$) is optimally forwarded to the cavity output port $\hat{U}_{\rm out}$, i.e., the output light is maximally squeezed if the cavity decay rate $\kappa$ equals $\tilde{\kappa}$.

[1–3] to even biology [4]. Squeezed states of light are also a key ingredient for continuous-variable information processing [5].

While the standard method for generating optical squeezing is to drive a nonlinear optical medium (see, e.g. [6]), it has long been realized [7] that squeezing can also be realized in optomechanical systems [8, 9], where cavity photons are coupled to mechanical motion by radiation pressure. The standard mechanism for such squeezing, termed ‘ponderomotive squeezing’ [7], relies on the mechanical resonator effectively mediating a (coherent) Kerr-type ($\chi_3$) optical nonlinearity [10, 11]; as in a Kerr medium, squeezing is produced by generating classical correlations between the amplitude and phase quadratures of light leaving the cavity. This sort of ponderomotive squeezing has recently been realized in experiments [12–14].

In this work, we describe a fundamentally different and potentially powerful new method for generating squeezed light using optomechanics, cf figure 1(a). Unlike standard ponderomotive squeezing, our scheme is not based on having the mechanics mediate a coherent (i.e., Hamiltonian) optical nonlinearity; instead, it uses the dissipative nature of the mechanical resonator. As we show, by using a (classical) bichromatic cavity drive, the mechanics can be made to mimic a dissipative squeezed reservoir. By careful tuning of the cavity laser drives, this effective mechanical reservoir acts as a ‘sink’ for the fluctuations of the incident light, and imprints its squeezed noise almost perfectly onto the output light (cf figure 1(b)). We also show that the squeezing generated in our approach can directly be used to enhance the intrinsic measurement sensitivity of the optomechanical cavity (i.e., to detect a signal coupled dispersively to the cavity). Note that although we focus on an optomechanical implementation of our scheme here, we stress that it could also be implemented using superconducting circuits [15–17] as our scheme relies only on having two modes coupled parametrically with both, beam-splitter and non-degenerate parametric amplifier terms.

Our scheme is well within the reach of current state-of-the-art optomechanical experiments, some of which have already made use of two-tone driving [18–21]. As we discuss, it has several advantages over standard ponderomotive squeezing. In particular, our scheme is efficient in the good-cavity limit commonly used in experiments, and squeezes the same quadrature of light over an appreciable bandwidth. This is to be contrasted against ponderomotive squeezing, which is not efficient in the good-cavity limit, and produces squeezing with a frequency-dependent squeezing angle. In addition, the squeezing generated in our scheme can be used directly to enhance cavity based measurements; one does not need to feed the squeezed light into a separate measurement device (see section 5).

Note that the scheme we describe is related to the protocol described and implemented by Wasilewski *et al* [22] to generate pulses of two-mode squeezed light. Their approach did not use mechanical interactions, but rather interactions with two polarized atomic spin-ensembles, each of which acts as an oscillator. While similar in spirit, there are some important differences: our scheme generates continuous-wave squeezed light, and makes use of dissipation in a fundamental way (in contrast, [22] does not treat atomic dissipation as it plays no role in their approach). Our scheme is also related to our earlier proposal for generating strong mechanical squeezing in an optomechanical cavity [23] (which in turn is related to [24] and earlier proposals [25–28]). Unlike that problem, the interest here is on generating squeezing of an output field (as opposed to an intracavity field); similar to the situation with squeezing via parametric processes [29, 30], there are crucial differences between these two goals.

## 2. Model

We consider a standard, single-sided optomechanical cavity, where electromagnetic radiation couples to mechanical motion via radiation pressure, cf figure 1(a) (non-ideal or two-sided cavities are discussed in the appendix). The optomechanical Hamiltonian reads [31]

$$\hat{H} = \hbar\omega_{\mathrm{cav}}\hat{a}^\dagger\hat{a} + \hbar\Omega\hat{b}^\dagger\hat{b} - \hbar g_0\left(\hat{b}^\dagger + \hat{b}\right)\hat{a}^\dagger\hat{a} + \hat{H}_{\mathrm{dr}}. \tag{1}$$

where $\omega_{\mathrm{cav}}$ ($\Omega$) is the cavity (mechanical) resonance frequency, $\hat{a}$ $\left(\hat{b}\right)$ the photon (phonon) annihilation operator and $g_0$ the optomechanical coupling strength. $\hat{H}_{\mathrm{dr}} = \hbar\left(\alpha\left(t\right)\hat{a}^\dagger + \mathrm{h.\,c.}\right)$ is the coherent laser driving Hamiltonian where $\alpha\left(t\right)$ describes a general, coherent multi-tone laser drive. In the following, we decompose the photon annihilation operator $\hat{a} = \bar{a} + \hat{d}$ into a classical amplitude $\bar{a}$ and quantum fluctuations $\hat{d}$. Treating cavity dissipation via standard input–output theory [32], the dynamics of the quantum fluctuations is given by the quantum Langevin equation

$$\dot{\hat{d}} = \frac{\mathrm{i}}{\hbar}\left[\hat{H}, \hat{d}\right] - \frac{\kappa}{2}\hat{d} - \sqrt{\kappa}\hat{d}_{\mathrm{in}}, \tag{2}$$

where $\kappa$ is the cavity decay rate. The equation of motion for the mechanical operator $\hat{b}$ reads

$$\dot{\hat{b}} = \frac{\mathrm{i}}{\hbar}\left[\hat{H}, \hat{b}\right] - \frac{\Gamma_M}{2}\hat{b} - \sqrt{\Gamma_M}\hat{b}_{\mathrm{in}},$$

where $\Gamma_M$ is the mechanical decay rate. The non-zero noise correlators read $\left\langle \hat{d}_{\text{in}}(t)\hat{d}_{\text{in}}^{\dagger}(t') \right\rangle = \delta(t-t')$, $\left\langle \hat{b}_{\text{in}}(t)\hat{b}_{\text{in}}^{\dagger}(t') \right\rangle = (n_{\text{th}}+1)\delta(t-t')$ and $\left\langle \hat{b}_{\text{in}}^{\dagger}(t)\hat{b}_{\text{in}}(t') \right\rangle = n_{\text{th}}\delta(t-t')$, where $n_{\text{th}}$ is thermal occupancy of the mechanical reservoir.

Our interest is on the noise properties of the light leaving the cavity. The fluctuations in the output light is described by $\hat{d}_{\text{out}}$, which in turn is determined by the incident noise $\hat{d}_{\text{in}}$ and the intracavity light $\hat{d}$ via the input–output relation $\hat{d}_{\text{out}} = \hat{d}_{\text{in}} + \sqrt{\kappa}\hat{d}$ [32]. A general quadrature of the output light is defined by

$$\hat{U}_{\varphi}^{\text{out}} = \left(\hat{d}_{\text{out}}e^{-\text{i}\varphi} + \hat{d}_{\text{out}}^{\dagger}e^{\text{i}\varphi}\right)\Big/\sqrt{2}. \tag{3}$$

The fluctuations in this quantity are quantified by the (measurable) spectral density:

$$S_{U_{\varphi}}^{\text{out}}[\omega] = \left\langle \int \text{d}\tau\, e^{\text{i}\omega\tau} \left\langle \hat{U}_{\varphi}^{\text{out}}(t+\tau/2)\hat{U}_{\varphi}^{\text{out}}(t-\tau/2) \right\rangle \right\rangle_t, \tag{4}$$

where $\langle\cdot\rangle_t$ denotes a time average over the center-of-mass time $t$ (i.e., we are interested in the stationary part of the noise).

If the output light is in a coherent state, $\hat{d}_{\text{out}}$ will be in its vacuum, and $S_{U_{\varphi}}^{\text{out}}[\omega] = 1/2 \equiv S_{SN}^{\text{out}}$ (i.e., the shot-noise value); with the optomechanical interaction, we will obtain deviations from this result. We will focus on the output quadrature exhibiting the minimum noise at a given frequency $\omega$, obtained by choosing the optimal angle $\varphi[\omega]$ (the squeezing angle). Defining the orthogonal quadratures $\hat{U}_1^{\text{out}} = \hat{U}_{\varphi=0}^{\text{out}}$ and $\hat{U}_2^{\text{out}} = \hat{U}_{\varphi=\pi/2}^{\text{out}}$, a straightforward optimization yields that the noise of this optimal quadrature is (see, e.g., [33])

$$S_{\text{opt}}^{\text{out}} = \frac{2S_{U_1}^{\text{out}}S_{U_2}^{\text{out}} - 2\left[S_{U_1U_2}^{\text{out}}\right]^2}{S_{U_1}^{\text{out}} + S_{U_2}^{\text{out}} + \sqrt{\left[S_{U_1}^{\text{out}} - S_{U_2}^{\text{out}}\right]^2 + 4\left[S_{U_1U_2}^{\text{out}}\right]^2}}. \tag{5}$$

Here, the cross-correlator $S_{U_1U_2}^{\text{out}}[\omega]$ measures the classical (i.e., symmetrized) correlations between $\hat{U}_1^{\text{out}}$ and $\hat{U}_2^{\text{out}}$, and is defined as:

$$S_{U_1U_2}^{\text{out}}[\omega] = \frac{1}{2}\left\langle \int \text{d}\tau\, e^{\text{i}\omega\tau} \left\langle \hat{U}_1^{\text{out}}(t+\tau/2)\hat{U}_2^{\text{out}}(t-\tau/2) + \hat{U}_2^{\text{out}}(t+\tau/2)\hat{U}_1^{\text{out}}(t-\tau/2) \right\rangle \right\rangle_t.$$

## 3. Ponderomotive squeezing

We begin by quickly reviewing the standard mechanism for optomechanical squeezed light generation, ponderomotive squeezing [7, 10–14], where one uses the *coherent* (i.e., Hamiltonian) optical nonlinearity induced by the coupling to the mechanical resonator. We assume a resonantly driven optomechanical cavity, i.e., $\alpha(t) = \alpha_L e^{-\text{i}\omega_{\text{cav}}t}$, where $\alpha_L$ is the laser amplitude. Going into an interaction picture with respect to the free cavity Hamiltonian and performing a standard linearization on (1) (i.e., dropping terms cubic in $\hat{d}$, $\hat{d}^\dagger$) one finds

$$\hat{H} = \hbar\Omega\hat{b}^\dagger\hat{b} - \sqrt{2}\,\hbar G\hat{U}_1\left(\hat{b} + \hat{b}^\dagger\right). \tag{6}$$

where $G = g_0\bar{a}$ is the drive-enhanced optomechanical coupling strength; without loss of generality, we take the average cavity amplitude $\bar{a}$ to be real. With this choice, $\hat{U}_1$ and $\hat{U}_2$ correspond respectively to standard amplitude and phase quadratures. Their fluctuations are given by [33]

$$S_{U_1}^{\rm out} = S_{\rm SN}^{\rm out} \quad \text{and} \quad S_{U_2}^{\rm out} = S_{U_1}^{\rm out} + 2\left[S_{U_1U_2}^{\rm out}\right]^2 + \delta S, \tag{7}$$

where

$$\delta S = \tilde{S}\left(\tilde{S} + \coth \hbar\omega/2k_BT\right),$$
$$\tilde{S} = 2\Omega G^2\kappa\,\mathrm{Im}\left\{\chi_M\right\}\big/\left(\kappa^2/4 + \omega^2\right),$$

and

$$\chi_M^{-1} = \Omega^2 - \omega^2 - \mathrm{i}\omega\Gamma_M$$

is the mechanical susceptibility.

Given that neither $U_1$ nor $U_2$ is squeezed, obtaining squeezing will necessarily require non-zero classical correlations between the amplitude and phase quadrature (i.e., $S_{U_1U_2}^{\rm out} \neq 0$), as follows from equations (5) and (7). These correlations are created by the mechanical motion. From the last term of equation (6), we see that the amplitude ($U_1$) fluctuations of the light are a driving force on the mechanics. The same term tells us that the resulting mechanical motion modulates the phase of the light leaving the cavity (i.e., the $U_2$ quadrature). One finds that the amplitude-phase correlator has a simple form which completely reflects this intuitive picture:

$$S_{U_1U_2}^{\rm out}[\omega] \propto \frac{4G^2}{\kappa}\frac{\Omega}{1 + (2\omega/\kappa)^2}\,\mathrm{Re}\left\{\chi_M[\omega]\right\}, \tag{8}$$

where $\omega$ is measured in our rotating frame (i.e., $\omega = 0$ corresponds to the cavity resonance). Note that only the real part of $\chi_M$ enters, as only in-phase correlations between $U_1$ and $U_2$ are relevant to squeezing (i.e., the correlations are induced by a *coherent* interaction only, since the dissipative part $\mathrm{Im}\left\{\chi_M\right\}$ of $\chi_M$ does *not* enter). Such in-phase correlations between amplitude and phase quadratures would naturally be created if we had a Kerr nonlinearity in the cavity, i.e., a term $\hat{a}^\dagger\hat{a}^\dagger\hat{a}\hat{a}$ in the cavity Hamiltonian. Thus, ponderomotive squeezing involves the optomechanical interaction mimicking the effects of a (instantaneous, coherent, Hamiltonian) Kerr interaction in the cavity. Note that the optomechanical interaction was recently compared to a Kerr nonlinearity also in [34, 35].

It thus follows that ponderomotive squeezing will be strongest at frequencies $\omega$, where the correlator $S_{U_1U_2}^{\rm out}$ is large; by combining equations (5) and (7) one finds $S_{\rm opt}^{\rm out} \propto 1\big/\left[S_{U_1U_2}^{\rm out}\right]^2$ for $S_{U_1U_2}^{\rm out} \gg 1$. The correlations will in turn be large when the real part of the mechanical susceptibility is large. This naturally occurs at the cavity resonance frequency (i.e., $\omega = 0$ in equation (8)), and also near (but not at) the mechanical sideband frequencies, i.e., frequencies

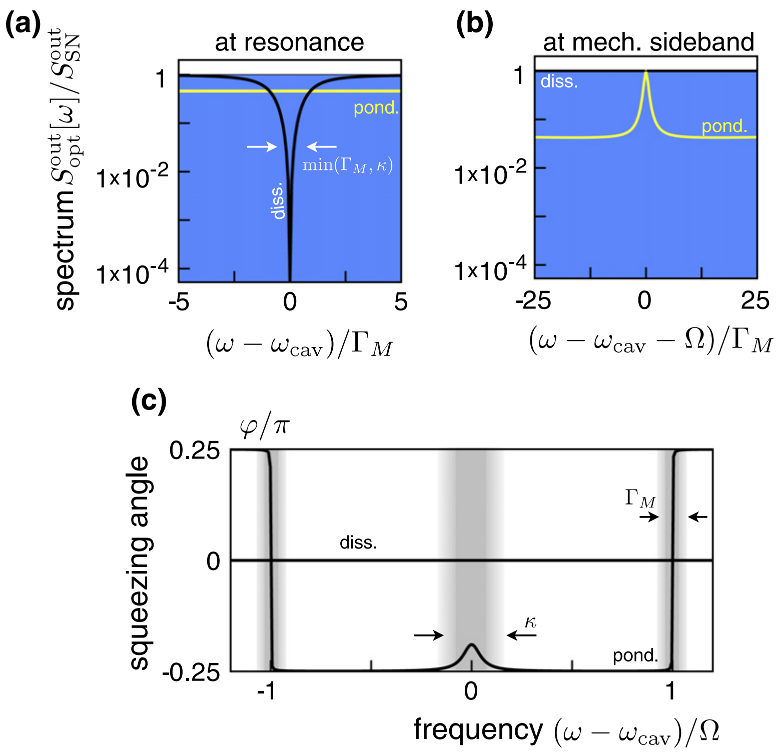


**Figure 2.** Ponderomotive squeezing versus dissipative squeezing spectra in the good cavity limit (where the impedance matching condition $\tilde{\kappa} = \kappa$ is assumed, cf main text). (a) Output light spectra versus frequency near the cavity resonance frequency $\omega_{\text{cav}}$ in the good cavity limit (where $\kappa/\Omega \to 0$ for our dissipative scheme and $\kappa/\Omega = 1/10$ for ponderomotive squeezing). The squeezing bandwidth at $\omega_{\text{cav}}$ is set by $\kappa$ for ponderomotive squeezing. For our dissipative scheme it is set by $\min\{\kappa, \Gamma_M\}$. Both schemes generate maximum squeezing at $\omega_{\text{cav}}$ within this bandwidth. However, our dissipative scheme outperforms ponderomotive squeezing in the good cavity limit. (b) Output light spectra at the mechanical sideband $\omega \approx \Omega$. Our dissipative scheme does not generate squeezing while ponderomotive squeezing does generate squeezing. (c) Squeezing angle $\varphi$ (cf equation (3)) versus frequency. For dissipative squeezing, the squeezing angle is constant for all frequencies. In contrast, the squeezing angle $\varphi_{\text{opt}}$ corresponding to optimal ponderomotive squeezing varies on a scale $\sim\kappa$ at the cavity resonance and $\sim\Gamma_M$ close to the mechanical sideband. (Parameters: (a) $\Gamma_M = 2 \times 10^{-5}\,\kappa$, $n_{\text{th}} = 10$, and $C = 10^5$. (b) Same as (a), $\tilde{\kappa} = 4\mathcal{G}^2/\Gamma_M = \kappa$. (c) Same as in (a).)

$\omega = \pm\Omega + \delta$ where $|\delta| \sim \Gamma_M$. Figure 2 shows this expected frequency dependence of ponderomotive squeezing.

It is often overlooked that the same intuition used above tells us that ponderomotive squeezing will be suppressed in the good-cavity limit $\kappa \ll \Omega$, a limit necessary for ground-state optomechanical cooling and other desirable optomechanical protocols. At the cavity resonance, $S^{\text{out}}_{U_1 U_2} \propto 4G^2/(\kappa\Omega)$, independent of the sideband parameter $\kappa/\Omega$ and mechanical damping rate $\Gamma_M$. Thus, in the limit $\Omega/\kappa \to \infty$ while $G/\kappa$ remains fixed, ponderomotive squeezing disappears at the cavity frequency. Indeed, we find $S^{\text{out}}_{\text{opt}}/S^{\text{out}}_{\text{SN}} \approx 1 - 16G^2/(\kappa\Omega)$ in this limit. The situation is different for frequencies close to the mechanical sidebands, i.e., $\omega = \pm\Omega + \delta$ with $|\delta| \sim \Gamma_M$. In the bad cavity limit $\kappa \gg \Omega$, $S^{\text{out}}_{U_1U_2} \propto C$, where the cooperativity $C = 4G^2/(\kappa\Gamma_M)$. Thus, squeezing close to the mechanical sideband and for $\kappa \gg \Omega$ is controlled by the cooperativity only. In the good cavity limit $\kappa \ll \Omega$, however, $S^{\text{out}}_{U_1U_2} \propto C(\kappa/\Omega)^2$. Thus, in the good cavity limit $\kappa/\Omega \to 0$, squeezed light cannot be generated effectively using the standard ponderomotive squeezing mechanism.

## 4. Dissipative output light squeezing

Given the general desirability of having optomechanical systems in the good-cavity limit (e.g. for cooling [36–40], state transfer [41–45], entanglement generation [46–50], etc), it would be extremely useful to find an alternative squeezing scheme which is efficient in this regime. To that end, we now introduce an approach which generates squeezed light by explicitly using the dissipative nature of the mechanical resonator.

### 4.1. Basic scheme

Unlike ponderomotive squeezing, the dissipative approach to optomechanical squeezing requires driving the cavity with two lasers, with frequencies corresponding to the red and blue mechanical sidebands (i.e., $\alpha(t) = \alpha_+ e^{-i(\omega_{\text{cav}}+\Omega)t} + \alpha_- e^{-i(\omega_{\text{cav}}-\Omega)t}$); the resulting average classical amplitude is $\bar{a}(t) = e^{-i\omega_{\text{cav}}t}\sum_{\sigma=\pm}\bar{a}_\sigma e^{-i\sigma\Omega t}$. We again write the basic optomechanical Hamiltonian of equation (1) in an interaction picture, now with respect to both the free cavity and mechanical resonator Hamiltonians. Introducing mechanical quadrature operators $\hat{X}_1 = (\hat{b}^\dagger + \hat{b})/\sqrt{2}$ and $\hat{X}_2 = i(\hat{b}^\dagger - \hat{b})/\sqrt{2}$, and linearizing the Hamiltonian in the usual way, we find $\hat{H} = \hat{H}_S + \hat{H}_{CR}$, where

$$\hat{H}_S = -\hbar(G_+ + G_-)\hat{U}_1\hat{X}_1 - \hbar(G_- - G_+)\hat{U}_2\hat{X}_2, \tag{9}$$

$$\hat{H}_{\text{CR}} = -\hbar\hat{d}^\dagger\left(G_+\hat{b}e^{-2i\Omega t} + G_-\hat{b}^\dagger e^{2i\Omega t}\right) + \text{h. c.}. \tag{10}$$

Here $G_\pm = g_0\bar{a}_\pm$ are the many-photon optomechanical couplings associated with each drive tone; we take $\bar{a}_+$, $\bar{a}_-$ to be real and positive without any loss of generality. The terms in $\hat{H}_S$ describe resonant interaction processes that will give rise to squeezing, while those in $\hat{H}_{\text{CR}}$ are deleterious non-resonant interaction terms. For physical transparency, we will start by discussing the extreme good cavity limit $\kappa \ll \Omega$, and thus ignore the effects of $\hat{H}_{\text{CR}}$. We will also take $G_- \geqslant G_+$, which ensures the stability of the linearized system.

If $G_+ = G_-$, $\hat{H}_S$ has the form of a quantum non-demolition (QND) interaction, as both the quadratures $U_1$ and $X_1$ commute with the Hamiltonian; such a regime can be used to make a back-action evading measurement of the mechanical quadrature $X_1$ [1, 51, 52]. For $G_+ \neq G_-$, the second term in $\hat{H}_S$ is non-zero, and the QND structure is lost. As we recently discussed [23], this regime can be extremely efficient for the generation of mechanical squeezing.

Given that equation (9) is symmetric under interchange of mechanical and cavity quadratures, one might naturally suspect that it can also be exploited to generate optical squeezing. We now show that this is indeed the case, even though in the optical case, we are interested in squeezing a quadrature of the *output* light field, not the intracavity field. As is well known, the relationship between intracavity and output field squeezing can be non-trivial [29, 30]. We show that this is also the case here.

### *4.2. Underlying mechanism*

We start by describing the basic mechanism which gives rise to squeezing here, considering the most interesting regime where $0 < G_- - G_+ \ll G_- + G_+$; for simplicity, we also first consider the case of a large mechanical damping rate $\Gamma_M \gg \kappa$. The first term in $\hat{H}_S$ (cf equation (9)) causes the mechanical resonator's $X_2$ quadrature to measure the cavity $U_1$ quadrature: in the relevant low-frequency limit, one finds

$$\hat{X}_2 = 2\frac{G_+ + G_-}{\Gamma_M}\hat{U}_1 + \frac{2}{\sqrt{\Gamma_M}}\hat{X}_2^{\text{in}}.$$

Thus, the measurement strength $\propto G_- + G_+$. This also demonstrates that dissipation is a necessary ingredient for $\hat{X}_2$ to measure the $\hat{U}_1$ quadrature. In contrast, the second term in $\hat{H}_S$ perturbs the measured quadrature $U_1$ by applying a weak force $\propto \left(G_- - G_+\right)\hat{X}_2$. However, as $X_2$ has measured $U_1$, this becomes a weak feedback force.

The result of these two operations is a net additional damping of the cavity $U_1$ quadrature at rate $\tilde{\kappa} = 4\mathcal{G}^2/\Gamma_M$ due to the optomechanical interaction, where $\mathcal{G}^2 = G_-^2 - G_+^2$. The mechanical resonator is thus acting like a dissipative bath for the cavity photons. One must also ask about the extra noise introduced into the cavity quadrature $U_1$ via the optomechanical coupling. As this only involves the weak second term in $\hat{H}_S$ ($\propto\left(G_- - G_+\right)$, cf equation (9)), this noise is extremely small, much smaller than the noise we would expect if $\tilde{\kappa}$ was produced by a zero-temperature dissipative bath. The net result is that the mechanical resonator acts as a squeezed bath for the cavity, damping the $U_1$ quadrature while adding almost no fluctuations. This directly causes optical squeezing. The situation is of course reversed if we now ask about the cavity $U_2$ quadrature. As the measurement and feedback roles of the two terms in $\hat{H}_S$ are reversed for $U_2$, its fluctuations are naturally enhanced by the effective mechanical bath.

### *4.3. Detailed calculation*

The above picture provides intuition for how the combination of the Hamiltonian $\hat{H}_s$ in equation (9) and mechanical damping gives rise to squeezing of the intracavity field: the mechanical resonator (via autonomous measurement and feedback operations) mimics the actions of squeezed dissipative reservoir coupled to the cavity. To understand how this basic mechanism affects the output noise of the cavity, we simply solve the linearized equations of motion describing our system (now without any assumption of a large $\Gamma_M$).

To present the solutions in a transparent manner, we first introduce the self-energy of the cavity photons due to the optomechanical interaction and the corresponding dressed cavity susceptibility:

$$\Sigma[\omega] = \frac{-\mathrm{i}\left(G_-^2 - G_+^2\right)}{-\mathrm{i}\omega + \Gamma_M/2} \equiv \mathrm{Re}\ \Sigma[\omega] - \mathrm{i}\tilde{\kappa}[\omega]/2.$$

The corresponding dressed cavity susceptibility (Green function) is then

$$\chi_{\mathrm{cav}}[\omega] = \frac{1}{-\mathrm{i}\omega + (\kappa/2) + \mathrm{i}\Sigma[\omega]}. \tag{11}$$

The output cavity quadrature operators are then found to be

$$\hat{U}_1^{\mathrm{out}}[\omega] = \left(\kappa\chi_{\mathrm{cav}}[\omega] - 1\right)\hat{U}_1^{\mathrm{in}}[\omega] - \sqrt{\kappa\Gamma_M}\chi_{\mathrm{cav}}[\omega]\sqrt{\tilde{\kappa}[\omega]}\,\hat{\xi}_1[\omega], \tag{12}$$

$$\hat{U}_2^{\mathrm{out}}[\omega] = \left(\kappa\chi_{\mathrm{cav}}[\omega] - 1\right)\hat{U}_2^{\mathrm{in}}[\omega] + \sqrt{\kappa\Gamma_M}\chi_{\mathrm{cav}}[\omega]\sqrt{\tilde{\kappa}[\omega]}\,\hat{\xi}_2[\omega]. \tag{13}$$

These input/output relations have the expected simple form for a cavity which is coupled both to a coupling port (coupling rate $\kappa$) and to an additional dissipative reservoir (coupling rate $\tilde{\kappa}[\omega]$). The coupling to the additional reservoir both modifies the cavity susceptibility, and results in new driving noises. The first term on the rhs of equations (12)–(13) corresponds to the contribution to the output field from vacuum noise incident on the cavity from the coupling port: there is both a promptly reflected contribution, and a contribution where this noise enters the cavity before being emitted. Note that these terms are completely phase insensitive, i.e., identical in form for any choice of optical quadrature.

More interesting are the second terms on the rhs of equations (12)–(13), which represent the noise contributions from the effective mechanical bath coupled to the cavity. One finds

$$\hat{\xi}_{1/2} = \frac{1}{\sqrt{\tilde{\kappa}[\omega]}}\frac{G_- \mp G_+}{-\mathrm{i}\omega + \Gamma_M/2}\hat{X}_{2/1}^{\mathrm{in}}[\omega].$$

We see immediately that this effective bath seen by the cavity appears squeezed (i.e., the noise in $\hat{\xi}_1$ is much less than that in $\hat{\xi}_2$) even if the intrinsic mechanical dissipation is in a simple thermal state.

With these equations, the route towards optimal squeezing at frequency $\omega$ is clear: one needs both to have $G_- - G_+$ be as small as possible (so that the $\hat{\xi}_j$ noises are as squeezed as possible), while at the same time fulfilling an impedance matching condition that makes the first terms in equations (12)–(13) vanish, i.e., $\kappa\chi_{\mathrm{cav}}[\omega] = 1$. Physically, this impedance matching simply means that all the incident optical vacuum fluctuations on the cavity are completely absorbed by the mechanical resonator, cf figure 1(b). At the cavity resonance frequency ($\omega = 0$), this corresponds to a simple matching of damping rates

$$\tilde{\kappa}[0] = \kappa \Longleftrightarrow \frac{4\left(G_-^2 - G_+^2\right)}{\Gamma_M} = \kappa. \tag{14}$$

We also see that regardless of the frequency we consider, the $U_1[\omega]$ optical quadrature is the optimally squeezed quadrature; this is simply because the squeezing angle of our effective mechanical bath is frequency independent.

### *4.4. Results*

Having explained the basic dissipative squeezing mechanism, we now present results for the amount of generated squeezing, again starting with the extreme good cavity limit $\Omega \gg \kappa$. The simplest regime here is the weak-coupling regime, where the effective coupling $\mathcal{G} = \sqrt{G_-^2 - G_+^2}$ is much smaller than $\max\left(\Gamma_M, \kappa\right)$. The output light is maximally squeezed at the cavity frequency, cf figure 2; the squeezing remains appreciable away from the cavity resonance over a 'squeezing bandwidth' set by $\max\left\{\kappa, \Gamma_M\right\}$. The amount of squeezing at the cavity resonance is given by

$$\frac{S_{U_1}^{\text{out}}[\omega = 0]}{S_{\text{SN}}^{\text{out}}} = \frac{4\kappa\tilde{\kappa}\left(1 + 2n_{\text{th}}\right)e^{-2r} + \left(\kappa - \tilde{\kappa}\right)^2}{\left(\kappa + \tilde{\kappa}\right)^2}, \tag{15}$$

where we have introduced the squeezing parameter $r$ via $\tanh r = G_+/G_-$, i.e., the ratio of laser drive amplitudes. Note that this expression is valid in the extreme good cavity limit $\kappa/\Omega \to 0$ for all values of $\kappa$, $\tilde{\kappa}$ and $r$. For a fixed squeezing parameter $r$, the noise in the $U_1$ quadrature interpolates between three simple limits. For $\tilde{\kappa} = 0$ or $\tilde{\kappa} \gg \kappa$, the noise of the effective mechanical resonator is completely reflected from the cavity, and hence the output quadrature noise is the just vacuum noise of the incident field. In contrast, if the impedance matching condition of equation (14) is satisfied, then the output optical noise is completely determined by the effective mechanical bath; it thus has the value $\left(1 + 2n_{\text{th}}\right)e^{-2r}$, reflecting the effective temperature of the squeezed $\hat{\xi}_1$ noise associated with the effective mechanical bath.

The above result then implies that for the optimal impedance-matched case (which also implies being in the assumed weak-coupling regime, cf appendix A.1), the squeezing of the cavity light at resonance behaves as

$$S_{U_1}^{\text{out}}[0]/S_{\text{SN}}^{\text{out}} = \left(1 + 2n_{\text{th}}\right)e^{-2r} \approx \frac{1 + 2n_{\text{th}}}{4C}, \tag{16}$$

where we have introduced the optomechanical cooperativity $C = 4G_-^2/\kappa\Gamma_M$, and in the last expression we assumed $C \gg 1$.

It is also interesting to consider the purity of the output light generated; not surprisingly, for the optimal impedance matched case, this purity is completely determined by the purity of the mechanical noise. Parameterizing the purity of the output light via an effective number of thermal quanta $n_{\text{eff}}$, i.e.,

$$\left(1 + 2n_{\text{eff}}[\omega]\right)^2 = 4S_{U_1^{\text{out}}U_1^{\text{out}}}[\omega]S_{U_2^{\text{out}}U_2^{\text{out}}}[\omega],$$

one finds $n_{\text{eff}} = n_{\text{th}}$ at the cavity frequency $\omega = 0$ and for $\tilde{\kappa} = \kappa$.

### *4.5. Dissipative versus ponderomotive squeezing*

Let us now compare our dissipative scheme to ponderomotive squeezing. Ponderomotive squeezing squeezes light by correlating the incident optical vacuum fluctuations using the *coherent* Kerr interaction mediated by the mechanical resonator. In contrast, our approach does not rely on correlating the incident optical vacuum fluctuation; rather, we replace these fluctuations by squeezed noise emanating from the mechanical resonator. As discussed, our

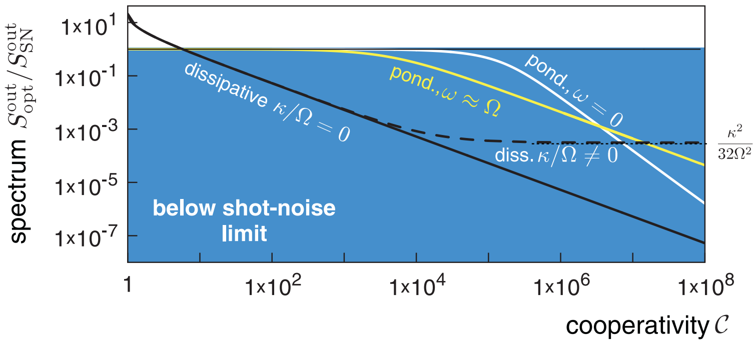


**Figure 3.** Dissipative versus ponderomotive squeezing in the good cavity limit. Black curves: output spectrum for the dissipative squeezing scheme at $\omega = 0$, i.e., at the cavity resonance frequency (black solid curve for $\kappa/\Omega = 0$ and black dashed curve for $\kappa/\Omega = 1/10$). White, yellow curves: optimized spectrum for standard ponderomotive squeezing, for both a frequency $\omega = 0$ and a frequency $\omega \sim \Omega$. Note that the value of the output spectrum for dissipative squeezing for small cooperativities is set by $n_{\mathrm{th}}$ (Parameters as in figures 2(a), (b)).

scheme also relies crucially on the *dissipative* nature of the mechanical resonator, i.e., on the imaginary part of the mechanical susceptibility $\chi_M$. In contrast, a non-vanishing $\mathrm{Im}\,\chi_M$ reduces the amount of ponderomotive squeezing, cf equation (7). We also note that our scheme is efficient in the good cavity limit and generates squeezing with a fixed squeezing angle, in contrast to ponderomotive squeezing.

Let us now turn to a *quantitative* comparison of our dissipative scheme to ponderomotive squeezing in the good cavity limit $\kappa \ll \Omega$, cf figure 3. We parametrize the red laser strength (or the resonant laser strength for ponderomotive squeezing) via the cooperativity $C = 4G_-^2/(\kappa\Gamma_M)$ (where $G_- \mapsto G$ for ponderomotive squeezing). For our dissipative scheme, we optimize the blue laser strength for any given cooperativity to fulfill the impedance matching condition (14).

We now compare the amount of squeezing generated by our dissipative scheme at the cavity frequency, i.e., $S^{\mathrm{out}}_{U_1}[0]$ to ponderomotive squeezing, i.e., to the optimized output light spectrum $S^{\mathrm{out}}_{\mathrm{opt}}$ at the cavity frequency and close to the mechanical sideband. For small cooperativities, $1 < C < (1 + n_{\mathrm{th}})^2/(1 + 2n_{\mathrm{th}})$, the output light spectrum in our scheme corresponds to thermally squeezed light (as $S^{\mathrm{out}}_{U_1}[0]/S^{\mathrm{out}}_{\mathrm{SN}} < (1 + 2n_{\mathrm{th}})$). As the squeeze parameter is small in this regime (cf equation (16)), $S^{\mathrm{out}}_{U_1}$ is larger than the shot noise value. In contrast, the output light spectrum $S^{\mathrm{out}}_{\mathrm{opt}}$ for ponderomotive squeezing in this small-$C$ case stays close to the shot-noise limit as $S^{\mathrm{out}}_{\mathrm{opt}} \approx S^{\mathrm{out}}_{\mathrm{SN}}$. As soon as $C \gtrsim n_{\mathrm{th}}/2$, our scheme generates *quantum* squeezed output light where $S^{\mathrm{out}}_{U_1}[0] < S^{\mathrm{out}}_{\mathrm{SN}}$. Ponderomotive squeezing, however, still stays close to the shot-noise limit, $S^{\mathrm{out}}_{\mathrm{opt}} \approx S^{\mathrm{out}}_{\mathrm{SN}}$. While increasing the cooperativity further, ponderomotive squeezing also starts to generate strong quantum squeezing, first close to the mechanical sideband, then also at $\omega = 0$. Thus, when comparing our scheme to ponderomotive squeezing for a fixed cooperativity, we see that our scheme outperforms ponderomotive squeezing in the good cavity limit. This can also be seen by studying the minimum cooperativity $C_{\min}$ needed to generate a certain amount of squeezing, e.g. 3 dB. For our scheme, we find $C_{\text{min}}^{\text{diss}} \gtrsim \left(1 + 2n_{\text{th}}\right)/2$. In contrast, for ponderomotive squeezing in the good cavity limit we find $C_{\text{min}}^{\text{ponderomotive squeezing}} \gtrsim \left(C_{\text{min}}^{\text{diss}} + \Omega/\left(\sqrt{2}\Gamma_M\right)\right)/4$. This is typically much larger than $C_{\text{diss}}$ since $\Gamma_M \ll \Omega$ for typical experiments.

### *4.6. Bad cavity effects on the generation of squeezed output light*

Up to now, we have focussed on the extreme good cavity limit, i.e., $\kappa/\Omega \to 0$. We now consider deviations that arise when $\kappa/\Omega$ is non-zero.

Thus, we now solve the full quantum Langevin equations including $\hat{H}_{\text{CR}}$ (i.e., no rotating-wave approximation) and analyze the output light spectrum $S_{U_1}^{\text{out}}[\omega]$. We find that the impedance matching condition $\tilde{\kappa} = \kappa$ still maximizes squeezing at the cavity frequency $\omega = 0$. Thus, we now compare $S_{U_1}^{\text{out}}[0]$ with and without bad cavity effects, cf figure 3. The amount of squeezing for moderate cooperativities does not differ from the good cavity prediction (16). As the cooperativity gets larger, however, the impact of bad cavity effects also becomes larger. As these terms tend to heat up the cavity quadrature non-resonantly, the maximum amount of squeezing our dissipative scheme can generate is limited. By taking $\hat{H}_{\text{CR}}$ into account up to leading order in $\kappa/\Omega$, we find that in the large cooperativity limit

$$\frac{S_{U_1}^{\text{out}}[0]}{S_{\text{SN}}^{\text{out}}} = \frac{\kappa^2}{32\Omega^2},$$

where $G_+/G_-$ was again chosen to fulfill the impedance matching condition (14).

## 5. Increasing the measurement sensitivity of an optomechanical cavity

As we have seen, our dissipative scheme can be used to generate squeezed output light. This light could then be fed into a separate measurement device to increase its measurement sensitivity. Such a scenario, however, involves two different devices which have to be coupled. In order to avoid unwanted coupling losses which could degrade the measurement sensitivity again or to keep the experiment as simple as possible, one might ask whether the squeezed light source and the measurement device could somehow be combined. In the following, we show that this is indeed possible: one could use the optomechanical cavity to both generate squeezed output light while increasing the sensitivity for measuring a dispersively-coupled signal at the same time.

### *5.1. Basic scheme*

We now consider an optomechanical cavity which is also dispersively coupled to a signal $z$ we want to measure (one could e.g. use an optomechanical setup in the microwave regime where a superconducting qubit is dispersively coupled to the microwave cavity; $z$ would then be a Pauli operator $\sigma_z$ for the qubit). We again assume two lasers driving the cavity on the red and blue mechanical sideband. As discussed above, the corresponding optomechanical interaction will cause the $\hat{U}_1^{\text{out}}$-quadrature to be squeezed at the cavity frequency. We now also add a resonant measurement tone which is used to probe the value of $z$. Thus,

$$\hat{H} = \hbar\omega_{\text{cav}}\hat{a}^\dagger\hat{a} + \hbar\Omega\hat{b}^\dagger\hat{b} - \hbar g_0\left(\hat{b}^\dagger + \hat{b}\right)\hat{a}^\dagger\hat{a} - \hbar A\hat{a}^\dagger\hat{a}\cdot z + \hat{H}_{\text{dr}},$$

where $\hat{H}_{\text{dr}} = \hbar\left(\alpha(t)\hat{a}^\dagger + \text{h. c.}\right)$ and $\alpha(t) = \alpha_+ e^{-\mathrm{i}(\omega_{\text{cav}}+\Omega)t} + \alpha_- e^{-\mathrm{i}(\omega_{\text{cav}}-\Omega)t} + \alpha_0 e^{-\mathrm{i}\omega_{\text{cav}}t}$. Note that the measurement tone at frequency $\omega_{\text{cav}}$ is spectrally very well resolved from the two tones at frequency $\omega_\pm = \omega_{\text{cav}} \pm \Omega$ used to generate squeezing. Thus, we expect the measurement tone to probe $z$ only without strongly degrading squeezing.

We now apply the displacement transformation $\hat{a} = \bar{a}(t) + \hat{d}$ with $\bar{a}(t) = e^{-\mathrm{i}\omega_{\text{cav}}t}\left(\sum_{\sigma=\pm}\bar{a}_\sigma e^{-\mathrm{i}\sigma\Omega t} + \mathrm{i}\bar{a}_0\right)$. We also assume $\bar{a}_i$ to be real. Note that the phase of the measurement tone is chosen such that the information of $z$ is imprinted in the squeezed quadrature, as we will see below. This is crucial to enhance the measurement sensitivity of the optomechanical cavity. We go into a rotating frame with respect to the free cavity and mechanical resonator Hamiltonian and apply standard linearization. We find $\hat{H} = \hat{H}_S + \hat{\hat{H}}_{\text{CR}}$ with

$$\hat{H}_S = -\hbar\left(G_+ + G_-\right)\hat{U}_1\hat{X}_1 - \hbar\left(G_- - G_+\right)\hat{U}_2\hat{X}_2 - \hbar\sqrt{2}A_0\hat{U}_2\cdot z, \tag{17}$$

and

$$\begin{aligned}\hat{\hat{H}}_{\text{CR}} = \hat{H}_{\text{CR}} &- 2\hbar G_0\left(\hat{X}_1\cos\Omega t + \hat{X}_2\sin\Omega t\right)\hat{U}_2\\ &- \sqrt{2}\hbar z\left[\left(A_+ + A_-\right)\hat{U}_1\cos\Omega t + \left(A_- - A_+\right)\hat{U}_2\sin\Omega t\right],\end{aligned} \tag{18}$$

where $\hat{H}_{\text{CR}}$ is given by equation (10). Here, $G_i = g_0\bar{a}_i$ is the driven-enhanced optomechanical coupling whereas $A_i = A\bar{a}_i$ is the driven-enhanced dispersive cavity–signal coupling. As in section 4.6, $\hat{\hat{H}}_{\text{CR}}$ represents non-resonant interaction terms that will have minimal effect in the $\kappa/\Omega \to 0$ limit.

### 5.2. Enhanced measurement rate

Let us first focus on the extreme good cavity limit and ignore $\hat{\hat{H}}_{\text{CR}}$. The last term in equation (17) implies that the $\hat{U}_1$ cavity quadrature measures $z$. Thus, the value of $z$ can be inferred by observing the output light quadrature $\langle\hat{I}\rangle = \sqrt{\kappa}\langle\hat{U}_1^{\text{out}}\rangle$ by using a homodyne measurement setup for instance.

As we are interested in a weak coupling between the cavity and the signal $z$, it will take a finite amount of time $\tau_{\text{meas}}$ to resolve the value of $z$ above the noise. This measurement time is quantified in the standard manner by the measurement time or rate $\Gamma_{\text{meas}} = 1/\tau_{\text{meas}}$ [32]. The measurement rate is related to the (zero frequency) susceptibility $\chi_{\text{meas}} \equiv \mathrm{d}\langle\hat{I}\rangle[\omega]/\mathrm{d}z$ and the symmetrized spectrum $\bar{S}_{II}$ of the homodyne current $\hat{I}$ at zero frequency via

$$\Gamma_{\text{meas}} = \frac{\chi_{\text{meas}}^2}{2\bar{S}_{II}[0]}. \tag{19}$$

Here, $\chi_{\text{meas}}[0]$ defines how much the average homodyne current changes when $z$ is statically changed and the symmetrized spectrum $\bar{S}_{II}$ quantifies the imprecision noise.

We now see the route towards an enhanced measurement rate: we simply need to use the optomechanical interaction and the consequent dissipative squeezing mechanism to squeeze $\hat{U}_1^{\text{out}}$ at zero frequency, and hence reduce $\bar{S}_{II}$ while keeping the measurement susceptibility $\chi_{\text{meas}}$ as large as possible.

As before, the optomechanical coupling in equation (17) generates squeezed output light where $\hat{U}_1^{\text{out}}$ is squeezed at $\omega = 0$, i.e., at the cavity frequency. This directly reduces the imprecision noise since $\bar{S}_{II}[\omega] = 2\kappa S_{U_1}^{\text{out}}[\omega]$ such that $\bar{S}_{II}\left[0\right] = \kappa\left(1 + 2n_{\text{th}}\right)e^{-2r}$ for the impedance matching condition $\tilde{\kappa} = \kappa$, cf equation (16). At the same time, the measurement susceptibility $\chi_{\text{meas}} = -\sqrt{2}\kappa A_0 \chi_{\text{cav}}\left[0\right]$ is not drastically changed. This is because when we optimally impedance match to maximize squeezing, i.e., choosing $\tilde{\kappa} = \kappa$, the optomechanical interaction only doubles the effective cavity damping, cf equation (11). Thus, $\chi_{\text{cav}}\left[0\right] = 1/\kappa$ is reduced only by a factor 1/2 as compared to the value one would obtain without the optomechanical interaction. Thus, we finally find

$$\Gamma_{\text{meas}} = \frac{A_0^2}{\kappa}\frac{e^{2r}}{1 + 2n_{\text{th}}}.$$

To quantify the sensitivity of our optomechanical cavity to $z$, we compare this measurement rate to the rate $\Gamma_{\text{meas}}^{\text{lc}}$ we expect when $z$ is measured using a linear cavity. This corresponds to turning off the optomechanical interactions in our scheme (i.e., $g_0 \to 0$). Hence, this comparison can be understood as being a benchmark for our dissipative squeezing scheme. We find

$$\frac{\Gamma_{\text{meas}}}{\Gamma_{\text{meas}}^{\text{lc}}} = \left(\frac{\chi_{\text{cav}}[0]}{\chi_{\text{cav}}^{\text{lc}}[0]}\right)^2 \frac{1}{S_{U_1}^{\text{out,diss}}/S^{\text{SN}}} \tag{20}$$

$$= \frac{e^{2r}}{4\left(1 + 2n_{\text{th}}\right)} \approx \frac{C}{1 + 2n_{\text{th}}}, \tag{21}$$

where the last term is valid in the large $C$ limit. Here, $\chi_{\text{cav}}$ is the dressed cavity susceptibility (cf equation (11)) and $\chi_{\text{cav}}^{\text{lc}}\left[0\right] = 2/\kappa$ is the susceptibility of a linear cavity at zero frequency. Thus, our scheme allows for an exponential enhancement of the measurement rate with the squeezing parameter $r$ (or a linear enhancement with cooperativity) as long as $C \gtrsim 1 + 2n_{\text{th}}$. For this comparison we have assumed equal decay rates $\kappa$ and the same read-out laser amplitudes.

The above analysis demonstrates that our dissipative optomechanical squeezing scheme can directly be used to enhance the intrinsic measurement sensitivity. The crucial trick allowing this direct enhancement is that our scheme generates squeezed output light without lowering the (dressed) cavity susceptibility drastically. Additionally, the cavity susceptibility is modified in a phase insensitive way, i.e., it is identical for all quadratures, cf (12), (13).

Note that it would be much more difficult to increase the intrinsic measurement sensitivity using ponderomotive squeezing: there, the optomechanical interaction effectively generates a Kerr-type optical nonlinearity [10, 11]. The corresponding linearized dynamics is similar to the dynamics of a parametric amplifier. Squeezing is generated by modifying the cavity susceptibility in a phase sensitive manner: one reduces the cavity response to vacuum noise for one quadrature while increasing the response for the conjugate quadrature. Reducing the response of the squeezed quadrature to noise, however, will also reduce its response to the signal $z$. Thus, the measurement rate $\Gamma_{\text{meas}}$ could be unchanged.

### 5.3. Influence of bad cavity effects on the measurement rate

Let us now discuss the influence of bad cavity effects on the measurement rate. Thus, we solve the quantum Langevin equations including $\hat{\hat{H}}_{\text{CR}}$ numerically and analyze the corresponding output light spectrum. Note first that the counter rotating terms which are independent of the mechanical resonator (cf second line of equation (18)) are a deterministic force driving the mean cavity quadratures only. As our system is linear, they thus have no impact on the noise properties or dressed cavity susceptibility, and thus play no role in the following discussion.

To gain an understanding of how bad cavity effects modify the measurement rate, let us first consider a very weak measurement tone, i.e., we focus on the limit $G_0 \approx 0$. In this case, $\hat{\hat{H}}_{\text{CR}} \approx \hat{H}_{\text{CR}}$. As discussed above, $\hat{H}_{\text{CR}}$ limits the maximum amount of squeezing, cf figure 3. However, squeezing is still given by equation (16) for moderate cooperativities. Thus, the measurement rate for weak measurement tones is still expected to scale like $e^{2r} \approx 4C$ until it is expected to saturate to $8\Omega^2/\kappa^2$ for larger cooperativities. Note that the assumption of a small $G_0$ does not necessarily imply a weak dispersive coupling $A_0$.

If we, however, were to increase the measurement tone strength further (e.g. to increase the absolute measurement rate $\Gamma_{\text{meas}} \propto A_0^2$), the additional counter-rotating term $\sim G_0$ in equation (18) becomes more and more important. This term is expected to further degrade the maximum achievable amount of squeezing, as the cavity $\hat{U}_2$ quadrature now gets additionally coupled to $\hat{X}_1$. In turn, it is expected to further limit the maximal achievable measurement rate. Thus, the favored strategy to generate an appreciable measurement rate $\Gamma_{\text{meas}}$ (as compared to $\Gamma_{\text{meas}}^{\text{lc}}$), hence, would be to keep $G_0$ as small as possible while aiming for a cooperativity which maximizes squeezing, and, hence, the measurement rate.

To verify our intuition, let us now focus on figure 4, where we depict the measurement rate enhancement factor $\Gamma_{\text{meas}}/\Gamma_{\text{meas}}^{\text{lc}}$ as a function of the red-laser driving strength and the measurement-tone strength. We choose the blue driving strength $G_+$ to optimize squeezing, i.e., to fulfill the impedance matching condition (14). We parametrize the red-laser strength via the cooperativity $C = 4G_-^2/(\kappa\Gamma_M)$. The measurement tone strength and, hence, also the strength of the unwanted optomechanical interaction induced by the measurement tone is parametrized via the measurement cooperativity $C_0 = 4G_0^2/(\kappa\Gamma_M)$. Note that as both $\Gamma_{\text{meas}}, \Gamma_{\text{meas}}^{\text{lc}} \sim A_0^2$, the measurement rate enhancement factor $\Gamma_{\text{meas}}/\Gamma_{\text{meas}}^{\text{lc}}$ is independent of the dispersive coupling $A_0$.

For a weak measurement tone $C_0 \ll 1$ we see that the ratio of the measurement rates $\Gamma_{\text{meas}}/\Gamma_{\text{meas}}^{\text{lc}}$ increases linearly with the cooperativity $C$ first until it saturates to $\sim 8\Omega^2/\kappa^2$ for large $C$. Thus, as expected, the unwanted optomechanical interaction induced by the measurement tone is negligible.

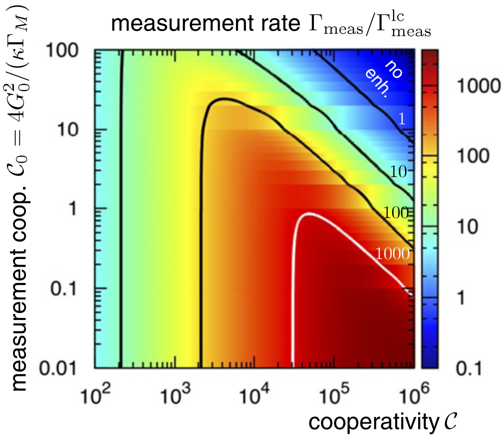


**Figure 4.** Enhancement $\Gamma_{\mathrm{meas}}/\Gamma^{\mathrm{lc}}_{\mathrm{meas}}$ of the dispersive measurement rate by dissipative squeezing. $\Gamma_{\mathrm{meas}}/\Gamma^{\mathrm{lc}}_{\mathrm{meas}}$ (cf equation (20)), i.e., the ratio of the squeezing enhanced measurement rate to the standard measurement rate (without optomechanical interaction), as a function of the cooperativity $C = 4G_-^2/(\kappa\Gamma_M)$ and the measurement tone driving strength (parametrized by the cooperativity $C_0 = 4G_0^2/(\kappa\Gamma_M)$). The black and white lines are contour lines depicting $\Gamma_{\mathrm{meas}}/\Gamma^{\mathrm{lc}}_{\mathrm{meas}} = 1, 10, 100, 1000$. (Parameters: $\Gamma_M = 2 \times 10^{-6}\,\Omega$, $\kappa = 0.05\,\Omega$, $\tilde{\kappa} = \kappa$ and $n_{\mathrm{th}} = 10$).

Let us now increase the measurement tone strength (i.e., $C_0$) further. For a fixed $C_0$, the measurement rate enhancement factor $\Gamma_{\mathrm{meas}}/\Gamma^{\mathrm{lc}}_{\mathrm{meas}}$ exhibits a maximum as a function of the cooperativity $C$ as the unwanted optomechanical interaction due to the measurement tone becomes important. Thus, an arbitrarily large cooperativity is *not* optimal in this regime. For realistic values of $C_0$, however, we still get a large maximum enhancement factor.

## 6. Conclusion

We have shown that strongly squeezed output light can be generated when an optomechanical cavity is driven by two lasers on the red and blue mechanical sideband. The output light is maximally squeezed when an impedance matching condition (cf equation (14)) is fulfilled. Then, all incident optical vacuum fluctuations are perfectly absorbed by the mechanical resonator and are replaced by effectively squeezed mechanical noise.

Furthermore, we have compared our dissipative scheme to ponderomotive squeezing and have shown that our dissipative scheme outperforms ponderomotive squeezing in the good cavity limit which is commonly used in experiments.

We also have shown that our dissipative scheme can directly be used to enhance the intrinsic measurement sensitivity of the optomechanical cavity. Thus, our scheme could e.g. be implemented in optomechanical setups working in the microwave regime to increase the measurement sensitivity of a dispersively coupled superconducting qubit. Note that although we have focussed on an optomechanical implementation of our scheme, it could also e.g. be implemented using superconducting circuits.

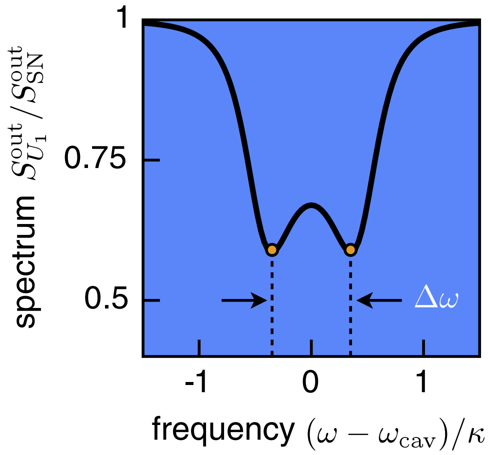


**Figure A.1.** Output light spectrum $S^{\rm out}_{U_1}$ in the 'strong coupling regime'. As the cavity photons and the mechanical mode are strongly coupled, two distinct minima are observed in the output light spectrum. Here, $\Delta\omega = 2\omega_+ = \sqrt{8\mathcal{G}^2 - \kappa^2 - \Gamma_M^2}\Big/\sqrt{2}$. (Parameters: $\Gamma_M/\kappa = 0.1$, $n_{\rm th} = 10$, $r = 5$ and $\mathcal{G} = 1/2$, i.e., $\mathcal{C} \approx 5.5 \times 10^3$ and $G_+/G_- \approx 1 - 9 \times 10^{-5}$.)

## Acknowledgments

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## Appendix

### *A.1. Dissipative squeezing in the strong coupling regime*

In this appendix we discuss the regime where the mechanical mode and the cavity are strongly coupled, i.e., when $\mathcal{G} = \sqrt{G_-^2 - G_+^2}$ is appreciable. In this case, we observe a normal mode splitting in the output light spectrum, cf figure A.1. It turns out that squeezing is maximized at frequencies

$$\omega_\pm = \pm \sqrt{8\mathcal{G}^2 - \kappa^2 - \Gamma_M^2}\Big/\left(2\sqrt{2}\right).$$

Thus, one enters the strong coupling regime if

$$8\mathcal{G}^2 \geqslant \kappa^2 + \Gamma_M^2. \tag{A.1}$$

Note that for impedance matched parameters $\tilde{\kappa} = 4\mathcal{G}^2/\Gamma_M = \kappa$, the strong coupling condition (A.1) cannot be fulfilled. Thus, for impedance matched parameters, squeezing is always maximized at the cavity resonance frequency.

Let us now briefly study how the maximum achievable squeezing at $\omega_\pm$ depends on the damping rates $\kappa$, $\Gamma_M$ and the coupling $\mathcal{G}$, i.e., we focus on the limit where the squeezing parameter $r \to \infty$. We find

$$S_{U_1}^{\text{out}} / S_{\text{SN}}^{\text{out}} \Big|_{\min} = \frac{(\Gamma_M - \kappa)^2 \left[ (\Gamma_M + \kappa)^2 - 16\mathcal{G}^2 \right]}{(\Gamma_M + \kappa)^2 \left[ (\Gamma_M - \kappa)^2 - 16\mathcal{G}^2 \right]}.$$

Thus, in the common limit where $\Gamma_M \ll \kappa$, one cannot generate squeezing dispersively in the strong coupling regime as $S_{U_1}^{\text{out}} / S_{\text{SN}}^{\text{out}} \Big|_{\min} \to 1$. If, however, $\Gamma_M = \kappa$, one is able to generate perfectly squeezed output light (i.e., $S_{U_1}^{\text{out}} / S_{\text{SN}}^{\text{out}} \Big|_{\min} = 0$) at frequencies $\omega_{\pm}$, *irrespective* of the size of $\mathcal{G}$.

### *A.2. Effects of intrinsic cavity losses and two-sided cavity*

In this appendix we focus on the dissipative generation of squeezed output light using a single-sided optomechanical cavity in the presence of internal losses. As we will see, internal losses will degrade the amount of squeezing generated and the state purity. In the presence of internal losses, the dynamics of the quantum fluctuations of the intracavity light field reads

$$\dot{\hat{d}} = \frac{i}{\hbar} \left[ \hat{H}, \hat{d} \right] - \frac{\kappa_{\text{tot}}}{2} \hat{d} - \sqrt{\kappa_O} \hat{d}_{\text{in}}^{(O)} - \sqrt{\kappa_I} \hat{d}_{\text{in}}^{(I)},$$

where $\kappa_{\text{tot}} = \kappa_O + \kappa_{\text{I}}$ is the total cavity decay rate, $\kappa_O$ is the photon decay rate through the output mirror and $\kappa_I$ is the rate with which photons decay internally (or e.g. through a second, unobserved mirror). As only the light leaving the cavity through the output mirror is of interest, we focus on the output light described by $\hat{U}_1^{\text{out}} = \left( \hat{d}_{\text{in}}^{\dagger(O)} + \hat{d}_{\text{in}}^{(O)} \right) / \sqrt{2}$ where $\hat{d}_{\text{out}}^{(O)} = \hat{d}_{\text{in}}^{(O)} + \sqrt{\kappa_O} \hat{d}$. For physical transparency, we assume the extreme good cavity limit, i.e., the system's Hamiltonian is given by equation (9). Solving the relevant equations of motion and calculating the output light spectrum $S_{U_1^{(O)}}^{\text{out}}[\omega]$ (cf equation (4)) we find that the output light quadrature $\hat{U}_1^{(0)}$ is still maximally squeezed at the cavity frequency if the impedance matching condition $\tilde{\kappa} = 4\mathcal{G}^2 / \Gamma_M = \kappa_{\text{tot}}$ is fulfilled. The amount of squeezing at $\omega = 0$ then reads

$$\begin{aligned} S_{U_1^{(o)}}^{\text{out}} / S_{\text{SN}}^{\text{out}} &= \frac{\kappa_I}{\kappa_{\text{tot}}} + \frac{\kappa_O}{\kappa_{\text{tot}}} \left( 1 + 2n_{\text{th}} \right) e^{-2r} \\ &\approx \frac{\kappa_I}{\kappa_{\text{tot}}} + \frac{\kappa_O}{\kappa_{\text{tot}}} \frac{1 + 2n_{\text{th}}}{4C}, \end{aligned}$$

where the last term is valid in the large $C$ limit. Thus, even if $C \to \infty$, one cannot squeeze the output light below $\kappa_I / \kappa_{\text{tot}}$.

Note that as a part of the light leaves the cavity into an unobserved mode, the purity of the squeezed output light is not given by $n_{\text{eff}}[0] = n_{\text{th}}[0]$ anymore. Instead, we find $n_{\text{eff}}[0] \sim \sqrt{\left( 1 + 2n_{\text{th}} \right) \kappa_O \kappa_I C / \kappa^2}$, i.e., the impurity increases without bound with the cooperativity $C$.

### *A.3. Effects of laser phase noise on dissipative squeezing of light*

In this appendix we discuss the impact of laser phase noise on our dissipative light squeezing scheme. Note that laser phase noise has already been studied in the context of e.g., optomechanical sideband cooling [53, 54], optomechanical entanglement [55, 56], and back-action evasion measurement schemes [57].

As before, we assume a two-tone driven optomechanical cavity, cf equation (1). However, we now also take a fluctuating laser phase $\varphi(t)$ into account, i.e., the laser drive now reads

$$\alpha(t) = \left(\alpha_+ e^{-\mathrm{i}\Omega t} + \alpha_- e^{\mathrm{i}\Omega t}\right) e^{-\mathrm{i}\omega_{\mathrm{cav}} t} e^{-\mathrm{i}\varphi(t)}.$$

Note that we have assumed a fixed *relative* phase between the two lasers. This implies that the maximally squeezed cavity output quadrature is independent of the laser phase noise.

To study the impact of the (global) fluctuating phase $\varphi(t)$ on the output light squeezing, we follow the analysis of laser phase noise presented in [55, 56]. Thus, we go into a *fluctuating* frame rotating at the fluctuating frequency $\omega_{\mathrm{cav}} + \dot{\varphi}(t)$, i.e., we perform the transformation

$$\hat{a}(t) \mapsto \hat{a}(t) \exp\left[-\mathrm{i}\omega_{\mathrm{cav}} t - \mathrm{i}\int_0^t \mathrm{d}\tau\, \dot{\varphi}(\tau)\right].$$

Note that this means that all optical quadratures have to be measured (e.g. in a homodyne setup) by using the same random phase noise $\varphi(t)$ as the local oscillator [55]. We now also go into an interaction picture with respect to the free mechanical resonator Hamiltonian. Applying again standard linearization, assuming $\dot{\varphi}\left(\bar{a}_\pm + \hat{d}\right) \approx \dot{\varphi}\bar{a}_\pm$ and applying a rotating wave approximation we finally find the equations of motion

$$\dot{\hat{U}}_1 = -\frac{\sqrt{2}}{g_0}\left(G_- - G_+\right)\dot{\varphi}\sin\Omega t - \left(G_- - G_+\right)\hat{X}_2 - \frac{\kappa}{2}\hat{U}_1 + \sqrt{\kappa}\hat{U}_1^{\mathrm{in}},$$
$$\dot{\hat{X}}_2 = \left(G_+ + G_-\right)\hat{U}_1 - \frac{\Gamma_M}{2}\hat{X}_2 + \sqrt{\Gamma_M}\hat{X}_2^{\mathrm{in}},$$

and

$$\dot{\hat{U}}_2 = \frac{\sqrt{2}}{g_0}\left(G_+ + G_-\right)\dot{\varphi}\cos\Omega t + \left(G_+ + G_-\right)\hat{X}_1 - \frac{\kappa}{2}\hat{U}_2 + \sqrt{\kappa}\hat{U}_2^{\mathrm{in}},$$
$$\dot{\hat{X}}_1 = -\left(G_- - G_+\right)\hat{U}_2 - \frac{\Gamma_M}{2}\hat{X}_1 + \sqrt{\Gamma_M}\hat{X}_1^{\mathrm{in}}.$$

Note that we again take $G_\pm$ to be real and positive, such that the maximally squeezed cavity output quadrature is $\hat{U}_1^{\mathrm{out}}$.

As dissipative light squeezing for impedance matched parameters $\tilde{\kappa} = 4\left(G_-^2 - G_+^2\right)/\Gamma_M = \kappa$ is strongest at the cavity resonance frequency (cf section 4.3), we now focus on the output light spectrum at the cavity frequency $\omega = 0$. Assuming (for simplicity) a flat spectrum for the laser phase noise, i.e., assuming $\left\langle \dot{\varphi}[\omega]\dot{\varphi}[\omega'] \right\rangle = 2\Gamma_L \delta(\omega + \omega')$ where $\Gamma_L$ is the laser linewidth, we find

$$S_{U_1}^{\mathrm{out}}[\omega = 0] \Big/ S_{\mathrm{SN}}^{\mathrm{out}} = \left(1 + 2n_{\mathrm{th}} + \Gamma_M \Gamma_L \big/ g_0^2\right) e^{-2r}. \tag{A.2}$$

By comparing equation (A.2) to our previous finding (16), we see that (global) laser phase noise effectively increases the mechanical bath temperature only. Thus, (global) phase noise is negligible if

$$\Gamma_L \ll g_0^2 \big/ \Gamma_M.$$

Note that this condition is equivalent to the one found in [54] which has to be fulfilled to be able to achieve optomechanical ground state cooling.

Thus, we conclude that (global) phase noise should not pose a strong limitation on our dissipative squeezing scheme.

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